The Chaotic Nature of Three-dimensional Magnetic Topology Revealed by Transversely Intersecting Invariant Manifolds

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Although adopted by Grad-Shafranov equation, EFIT, VMEC, *etc.*, the nested closed flux surface assumption does not necessarily hold when the axial symmetry of magnetic field is absent. An abundant amount of research has focused on how to stimulate a stochastic field layer at the plasma boundary to mitigate destructive type-I edge localized modes [1]. With regard to the influence of topology change on the scrape-off layer, a spiral ribbon-like pattern of heat deposition has also been reported and investigated in experiments. Based on the theory of dynamical system and chaos [2], we formalized relevant notions concerning magnetic topology and revealed the global structure of three-dimensional magnetic field. *The invariant manifold growth formula in cylindrical coordinates* is deduced and essential to determine the chaotic field regions (used to be called stochastic field), which induce a mixing effect inside plasma. It is proposed that the well-known notion of the last closed flux surface is substituted by more accurate invariant manifolds of the outmost hyperbolic cycle(s). The transverse intersection of invariant manifolds is a signature of chaos, indicating the intrinsic unpredictability (of field line tracing) in the long run. Having acquired the analytical form of invariant manifolds, we further regard the whole magnetic field as a functional argument of Poincaré map and utilize the functional derivative from functional analysis to obtain *the X-point shift under perturbation ($\delta\mathscr{B}$) formula*. Undoubtedly, the most important perturbation field is the derivative of magnetic field itself *w.r.t.* time, *i.e.* $\partial\mathscr{B}/\partial t$, giving the shift velocity of X-points when $\delta\mathscr{B}$ is substituted for $\partial\mathscr{B}/\partial t$ in the formula above.

In conclusion, a systematic analytic theory of three-dimensional magnetic fields has been established to facilitate comprehending the field structure and to provide guidance on control.

References

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